

Degree based Topological Indices of Linear Phenylene

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ABSTRACT: In this Paper, we obtained the vertex degree based topological indices Randic, Geometric – Arithmetic, Sum Connectivity, Harmonic, First Zagreb, Second Zagreb, Second Modified Zagreb, Inverse Sum, Alberston, Atom – bond Connectivity, Symmetric – Division index and Augmented Zagreb indices for Linear Phenylene. The vertex degree based topological indices for the Linear Phenylene are computed as a functional equations in ‘a’ and ‘n’, where ‘a’ represents the number of hexagons and ‘n’ represents the number of segments. The indices values for particular values of ‘a’ and ‘n’ are calculated and tabulated as a ready reckoner.

KEYWORDS: Vertex Degree, Topological Indices, Linear phenylene.

Mathematics Subject Classification: 05C05, 05C12

I. INTRODUCTION

A topological index of a chemical compound is an integer, derived following a certain rule, which can be used to characterize the chemical compound and predict certain physiochemical properties like boiling point, Molecular weight, density, refractive index, and so forth [1,2]. A Molecular graph $G = (V, E)$ is a simple graph having $n = |V|$ vertices and $m = |E|$ edges. The vertices $v \in V_i$ represent non hydrogen atoms and the edges $(v_i, v_j) \in E$ represent covalent bond between the corresponding atoms. In particular, hydrocarbons are formed only by carbon and hydrogen atom and their molecular graphs represent the carbon skeleton of the molecule. Molecular graphs are special type of chemical graphs, which represent the constitution of molecules. They are also called constitutional graphs. When the constitutional of molecules is represented in two dimensional basis, it is called structural graph. All molecular graphs considered in this paper are finite, Connected, loop less, and without multiples edges. Let $G = (V, E)$ be a graph with n vertices and m edges. The degree of a vertex $u \in V(G)$ is denoted by d_u and d_v are the number of vertices u and v respectively. [3,4] established simple relation between the second order Randic index of a phenylene and of the corresponding hexagonal squeeze. There are many topology indices defined on the basis of the vertex degree of graphs one of the degree based index namely harmonic index $H(G)$ [5,6]. Recently, on the occasion of the 30th anniversary of the first Zagreb indices a paper [7] was published in which their main properties were summarised. The Zagreb coincides were recently studied in some detailed and the relation between Zagreb indices and coincides were also examined. However, both the First Zagreb index and the second Zagreb index give greater Weights to the inner vertices and edges, the smaller weight to outer vertices and edges which opposes intuitive reasoning [8]. For a simple

Connected graph G , ${}^m M_1(G) = \sum_{v \in V(G)} \frac{1}{(d(v))^2}$ which was called the First Modified Zagreb index,

${}^m M_2(G) = \sum_{uv \in E(G)} \frac{1}{d(u)d(v)}$ which was called the Second Modified Zagreb index, where $d(u)$ and $d(v)$ are the

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degrees of vertices u and v respectively. Similarly all the other vertex degree based topological indices like Randic, Geometric – Arithmetic, Sum Connectivity, Harmonic, First Zagreb, Second Zagreb, Second Modified Zagreb, Inverse Sum, Alberston, Atom – bond Connectivity, Symmetric – Division index and Augmented Zagreb indices for Linear Phenylene are defined in theorems. Their notation and definitions are tabulated where ‘ i ’ represents d_u and ‘ j ’ represents $d_v, uv \in E(G)$. In this paper, we consider linear Phenylene denoted as $G(a,k,n)$ where ‘ a ’ represents number of hexagons in one segment, k represents number of squares connecting any two segments and n represents number of segments.

II. CASE 1: $G(1,1,n)$



In $G(a,k,n)$, let ‘ n ’ be the Number of segments, for any ‘ n ’ and $a = k$ for $n \geq 2$
 \Rightarrow Number of hexagons = $n_h = an$
 \Rightarrow Number of squares = $n_s = a(n - 1)$

For $G(1,0,1)$, (i.e) for $a=1, k=0, n=1$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 0, the number of edges with end degrees (3,3) is 0, total number of edges for $n=1$ is 6.

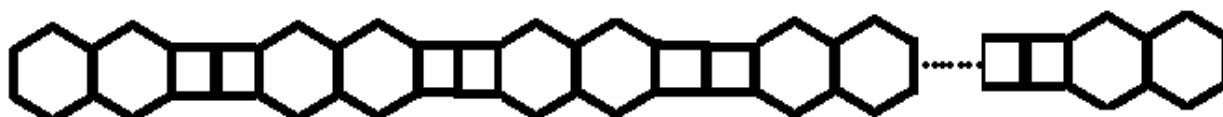
For $G(1,1,2)$, (i.e) for $a=1, k=1, n=2$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 4, the number of edges with end degrees (3,3) is 4, total number of edges for $n=2$ is 14.

For $G(1,1,3)$, (i.e) for $a=1, k=1, n=3$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 8, the number of edges with end degrees (3,3) is 8, total number of edges for $n=3$ is 22.

For $G(1,1,4)$, (i.e) for $a=1, k=1, n=4$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 12, the number of edges with end degrees (3,3) is 12, total number of edges for $n=4$ is 30.

For $G(1,1,n)$ for $a=1, k=1$ for $n \geq 1$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is $4n-4$, the number of edges with end degrees (3,3) is $4n-4$, total number of edges for any n is given by $8n-2$.

III. CASE 2: $G(2,2,n)$



For $G(2,0,1)$, (i.e) for $a=2, k=0, n=1$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 4, the number of edges with end degrees (3,3) is 1, total number of edges for $n=1$ is 11.

For $G(2,2,2)$, (i.e) for $a=2, k=2, n=2$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 12, the number of edges with end degrees (3,3) is 9, total number of edges for $n=2$ is 27.

For $G(2,2,3)$, (i.e) for $a=2, k=2, n=3$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 20, the number of edges with end degrees (3,3) is 17, total number of edges for $n=3$ is 43.

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For $G(2,2,4)$, (i.e) for $a=2, k=2, n=4$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 28, the number of edges with end degrees (3,3) is 25, total number of edges for $n=59$.

For $G(2,2,n)$, (i.e) for $a=2, k=2$ for $n \geq 1$ the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is $8n-4$, the number of edges with end degrees (3,3) is $8n-7$, total number of edges for any n is $16n-5$.

IV. CASE 3:G(3,3,n)

In $G(a,k,n)$, let 'n' be the Number of segments, for any 'n' and $a=k$ for $n \geq 2$

For $G(3,0,1)$, (i.e) for $a=3, k=0, n=1$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 8, the number of edges with end degrees (3,3) is 2, total number of edges for $n=1$ is 16.

For $G(3,3,2)$, (i.e) for $a=3, k=3, n=2$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 20, the number of edges with end degrees (3,3) is 14, total number of edges for $n=2$ is 40.

For $G(3,3,n)$, (i.e) for $a=3, k=3, n=3$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 32, the number of edges with end degrees (3,3) is 26, total number of edges for $n=3$ is 64.

For $G(3,3,n)$, (i.e) for $a=3, k=3$ for $n \geq 1$ the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is $12n-4$, the number of edges with end degrees (3,3) is $12n-10$, total number of edges for any n is $24n-8$.

V. CASE 4:G(4,4,n)

For $G(4,4,n)$, (i.e) for $a=4, k=0, n=1$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 12, the number of edges with end degrees (3,3) is 3, total number of edges for $n=1$ is 21.

For $G(4,4,n)$, (i.e) for $a=4, k=1, n=2$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 28, the number of edges with end degrees (3,3) is 19, total number of edges for $n=2$ is 53.

For $G(4,4,n)$, (i.e) for $a=4, k=2, n=3$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 44, the number of edges with end degrees (3,3) is 35, total number of edges for $n=3$ is 85.

For $G(4,4,n)$, (i.e) for $a=4, k=3, n=4$, the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is 60, the number of edges with end degrees (3,3) is 51, total number of edges for $n=4$ is 111.

For $G(4,4,n)$, (i.e) for $a=3, k=3$ for $n \geq 1$ the number of edges with end degrees (2,2) is 6, the number of edges with end degrees (2,3) is $16n-4$, the number of edges with end degrees (3,3) is $16n-13$, total number of edges for any n is given by $32n-11$.

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VI. GENERALIZATION FOR $G(a,k,n)$

For $a=1$ then $k=0$ for $n = 1$ and $a = k$ for $n \geq 2$, the results obtained from case 1 to case 4 are tabulated as follows:

From the results of Case 1:

a	K	n	Number of edges		Number of edges		Number of edges	
			i=2	j=2	i=2	j=3	i=3	j=3
1	0	1	6		0		0	
1	1	2	6		4		4	
1	1	3	6		8		8	
1	1	4	6		12		12	
Generalizations			6		$4n-4$		$4n-4$	

From the results of Case 2:

a	k	n	Number of edges		Number of edges		Number of edges	
			i=2	j=2	i=2	j=3	i=3	j=3
2	0	1	6		4		1	
2	2	2	6		12		9	
2	2	3	6		20		17	
2	2	4	6		28		25	
Generalizations			6		$8n-4$		$8n-7$	

From the results of Case 3:

a	K	n	Number of edges		Number of edges		Number of edges	
			i=2	j=2	i=2	j=3	i=3	j=3
3	0	1	6		8		2	
3	3	2	6		20		14	
3	3	3	6		32		26	
3	3	4	6		44		38	
Generalizations			6		$12n-4$		$12n-10$	

From the results of Case 4:

a	K	n	Number of edges		Number of edges		Number of edges	
			i=2	j=2	i=2	j=3	i=3	j=3
4	0	1	6		12		3	
4	4	2	6		28		19	
4	4	3	6		44		35	
4	4	4	6		60		51	
Generalizations			6		$16n-4$		$16n-13$	

Hence we obtained the formula for linear phenylene $G(a, k, n)$ for all $n, a \geq 1$ the number of edges with end degrees (2,2) is equal to 6, the number of edges with end degrees (2,3) is $(4n - 4)$, the number of edges with end degrees (3,3) is given by $4an - (3a + 1)$.

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M-Polynomial of linear Phenylene is developed and it is given by

$$G(a, k, n) = 6x^2y^2 + (4an - 4)x^2y^3 + (4an - (3a + 1))x^3y^3 \text{ for all } n, a \geq 1, k \geq 0$$

Using the M-Polynomial of Linear phenylene the following theorems are established.

Theorem:1

Let $G(a, k, n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Randic index of linear

phenylene is given by $\chi(G) = 2\sqrt{\frac{2}{3}}an + \frac{4an}{3} - a - 2\sqrt{\frac{2}{3}} + \frac{8}{3}$ for $n, a \geq 1, k \geq 0$.

Proof:

$$\text{For } n, a \geq 1, k \geq 0, \text{ Randic Index } \chi(G) = \frac{1}{\sqrt{ij}}$$

$$\begin{aligned} \chi(G) &= 6 \cdot \frac{1}{\sqrt{4}} + (4an - 4) \cdot \frac{1}{\sqrt{6}} + (4an - (3a + 1)) \cdot \frac{1}{\sqrt{9}} = \frac{6}{2} + \frac{4an}{\sqrt{6}} - \frac{4}{\sqrt{6}} + \frac{4an}{3} - \frac{3a}{3} - \frac{1}{3} \\ &= \frac{8}{3} + \frac{1}{\sqrt{6}}(4an - 4) + \frac{1}{3}(4an - 3a) \Rightarrow \chi(G) = 2\sqrt{\frac{2}{3}}an + \frac{4an}{3} - a - 2\sqrt{\frac{2}{3}} + \frac{8}{3} \end{aligned}$$

Theorem:2

Let $G(a, k, n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Geometric-Arithmetic index

of linear phenylene is given by $GA(G) = 4an + \frac{2\sqrt{6}}{5}(4an - 4) - 3a + 5$ for $n, a \geq 1, k \geq 0$.

Proof:

$$\text{For } n, a \geq 1, k \geq 0, \text{ Geometric - Arithmetic Index } GA(G) = \frac{2\sqrt{ij}}{i + j}$$

$$GA(G) = 6 \cdot \frac{2\sqrt{4}}{2+2} + (4an - 4) \frac{2\sqrt{6}}{2+3} + (4an - (3a + 1)) \frac{2\sqrt{9}}{3+3} = 4an + \frac{2\sqrt{6}}{5}(4an - 4) - 3a + 5.$$

Theorem:3

Let $G(a, k, n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Sum-Connectivity index of

linear phenylene is given by $SCI(G) = \frac{4an}{\sqrt{5}} + 2\sqrt{\frac{2}{5}}an - \sqrt{\frac{3}{2}}a - \frac{1}{\sqrt{6}} - \frac{4}{\sqrt{5}} + 3$ for $n, a \geq 1, k \geq 0$.

Proof:

$$\text{For } n, a \geq 1, k \geq 0, \text{ Sum - Connectivity } SCI(G) = \frac{1}{\sqrt{i + j}}$$

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$$\begin{aligned}
 SCI(G) &= 6 \cdot \frac{1}{\sqrt{4}} + (4an - 4) \frac{1}{\sqrt{5}} + (4an - (3a + 1)) \frac{1}{\sqrt{6}} = 3 + (4an - 4) \frac{1}{\sqrt{5}} + (4an - 3a - 1) \frac{1}{\sqrt{6}} \\
 &= \frac{4an}{\sqrt{5}} + 2\sqrt{\frac{2}{5}}an - \sqrt{\frac{3}{2}}a - \frac{1}{\sqrt{6}} - \frac{4}{\sqrt{5}} + 3
 \end{aligned}$$

Theorem:4

Let $G(a, k, n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Harmonic index of linear phenylene is given by $HI(G) = \frac{1}{15}(44an - 15a + 16)$ for $n, a \geq 1, k \geq 0$

Proof:

For $n, a \geq 1, k \geq 0$, Harmonic index $HI(G) = \frac{2}{i+j}$

$$\begin{aligned}
 HI(G) &= 6 \cdot \frac{2}{2+2} + (4an - 4) \frac{2}{2+3} + (4an - (3a + 1)) \frac{2}{3+3} \\
 &= 3 + \frac{8an}{5} - \frac{8}{5} + \frac{4an}{3} - \frac{3a}{3} - \frac{1}{3} = \frac{8an}{5} + \frac{4an}{3} - \frac{3a}{3} - \frac{1}{3} - \frac{8}{5} + 3 = \frac{44an}{15} - a + \frac{16}{15} \\
 HI(G) &= \frac{1}{15}(44an - 15a + 16)
 \end{aligned}$$

Theorem:5

Let $G(a, k, n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then First Zagreb index of linear phenylene is given by $M_1(G) = \{44an - 18a - 2\}$ for $n, a \geq 1, k \geq 0$.

Proof:

For $n, a \geq 1, k \geq 0$, First Zagreb Index $M_1(G) = i + j$

$$\begin{aligned}
 M_1(G) &= 6 \cdot 4 + (4an - 4)5 + (4an - (3a + 1))6 = 24 + (4an - 4)(5) + (4an - 3a - 1)(6) \\
 &= 24 + 20an - 20 + 24an - 18a - 6 = 20an + 24an - 18a - 20 - 6 + 24 = 44an - 18a - 2 \\
 M_1(G) &= 44an - 18a - 2
 \end{aligned}$$

Theorem:6

Let $G(a, k, n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Second Zagreb index of linear phenylene is given by $M_2(G) = 3(20an - 9a - 3)$ for $n, a \geq 1, k \geq 0$

Proof:

For $n, a \geq 1, k \geq 0$ by Second Zagreb index $M_2(G) = ij$

$$\begin{aligned}
 M_2(G) &= 6(2 * 2) + (4an - 4)(2 * 3) + (4an - (3a + 1))(3 * 3) \\
 &= 24 + (4an - 4)(6) + (4an - 3a - 1)(9) = 24 + 24an - 24 + 36an - 27a - 9 = 60an - 27a - 9 \\
 &= 3(20an - 9a - 3) \Rightarrow M_2(G) = 3(20an - 9a - 3)
 \end{aligned}$$

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Theorem:7

Let $G(a,k,n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Second Modified Zagreb

index of linear phenylene is given by $M_3(G) = \frac{1}{18}(20an - 6a + 13)$ for $n, a \geq 1, k \geq 0$

Proof:

For $n, a \geq 1, k \geq 0$, Second Modified Zagreb index $M_3(G) = \frac{1}{ij}$

$$M_3(G) = 6 \cdot \frac{1}{4} + (4an - 4) \cdot \frac{1}{6} + (4an - (3a + 1)) \cdot \frac{1}{9} = \frac{6}{4} + \frac{4an - 4}{6} + \frac{(4an - 3a - 1)}{9}$$

$$= \frac{3}{2} - \frac{4}{6} - \frac{1}{9} + \frac{4an}{6} + \frac{4an}{9} - \frac{3a}{9} = \frac{78}{108} + \frac{60an}{54} - \frac{3a}{9} = \frac{13}{18} + \frac{10an}{9} - \frac{a}{3} \Rightarrow M_3(G) = \frac{1}{18}(20an - 6a + 13)$$

Theorem:8

Let $G(a,k,n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Inverse sum index of linear

phenylene is given by $IS(G) = a \left(\frac{54n}{5} - \frac{9}{2} \right) - \frac{3}{10}$ for $n, a \geq 1, k \geq 0$.

Proof:

For $n, a \geq 1, k \geq 0$, Inverse Sum Index $IS(G) = \frac{ij}{i+j}$

$$IS(G) = 6 \cdot \frac{4}{2+2} + (4an - 4) \cdot \frac{6}{2+3} + (4an - (3a + 1)) \cdot \frac{9}{3+3} = \frac{24}{4} + (4an - 4) \frac{6}{5} + (4an - 3a - 1) \frac{9}{6}$$

$$= 6 + \frac{24an}{5} - \frac{24}{5} + \frac{36an}{6} - \frac{27a}{6} - \frac{9}{6} = 6 - \frac{24}{5} - \frac{3}{2} + \frac{144an + 180an}{30} - \frac{9a}{2} = \frac{-3}{10} + \frac{324an}{30} - \frac{9a}{2}$$

$$= \frac{54an}{5} - \frac{9a}{2} - \frac{3}{10} \quad IS(G) = a \left(\frac{54n}{5} - \frac{9}{2} \right) - \frac{3}{10}$$

Theorem:9

Let $G(a,k,n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Albertson index of linear

phenylene is given by $Alb(G) = \{4(an - 1)\}$ for $n, a \geq 1, k \geq 0$.

Proof:

For $n, a \geq 1, k \geq 0$ by Albertson index $Alb(G) = |i - j|$

$$Alb(G) = 6 \cdot 0 + (4an - 4) \cdot 1 + (4na - (3a + 1)) \cdot 0$$

$$Alb(G) = 4(an - 1)$$

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Theorem:10

Let $G(a,k,n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Atom-Bond connectivity

index of linear phenylene is given by $ABC(G) = 2\sqrt{2}an + \frac{8an}{3} - 2a + \sqrt{2} - \frac{2}{3}$ for $n, a \geq 1, k \geq 0$.

Proof:

For $n, a \geq 1, k \geq 0$, Atom Bond Connectivity $ABC(G) = \sqrt{\frac{i+j-2}{ij}}$

$$\begin{aligned} ABC(G) &= 6 \cdot \sqrt{\frac{2}{4}} + (4an - 4) \cdot \sqrt{\frac{3}{6}} + (4an - (3a + 1)) \cdot \sqrt{\frac{4}{9}} \\ &= \frac{6\sqrt{2}}{\sqrt{4}} + \frac{4\sqrt{3}an}{\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}} + \frac{4\sqrt{4}an}{\sqrt{9}} - \frac{3\sqrt{4}a}{\sqrt{9}} - \frac{\sqrt{4}}{\sqrt{9}} = 3\sqrt{2} + \frac{4an - 4}{\sqrt{2}} + \frac{2}{3}(4an - 3a - 1) \\ &= 2\sqrt{2}an + \frac{8an}{3} - 2a + \sqrt{2} - \frac{2}{3} \end{aligned}$$

Theorem:11

Let $G(a,k,n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Symmetric Division Index

of linear phenylene is given by $SD(G) = \frac{4}{3} + a \left(\frac{50n}{3} - 6 \right)$ for $n, a \geq 1, k \geq 0$

Proof:

For $n, a \geq 1, k \geq 0$ by Symmetric Division Index $SD(G) = \frac{i^2 + j^2}{ij}$

$$\begin{aligned} SD(G) &= 6 \cdot \frac{8}{4} + (4an - 4) \cdot \frac{13}{6} + (4an - (3a + 1)) \cdot \frac{18}{9} = 10 - \frac{52}{6} + \frac{52an}{6} + 8an - 6a \\ &= \frac{8}{6} + \frac{52an + 48an}{6} - 6a \\ SD(G) &= \frac{4}{3} + a \left(\frac{50n}{3} - 6 \right) \end{aligned}$$

Theorem:12

Let $G(a,k,n)$ be the linear phenylene, 'a' represents the number of hexagons in a segment and 'k' represents the number of Squares connecting any two segment, 'n' represents the number of segments in G , then Augmented Zagreb index of

linear phenylene is given by $AZI(G) = \frac{295}{64} + a \left(\frac{1241n}{16} - \frac{2187}{64} \right)$ for $n, a \geq 1, k \geq 0$

Proof:

For $n, a \geq 1, k \geq 0$, Augmented Zagreb index $AZI(G) = \left(\frac{ij}{i+j-2} \right)^3$

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$$AZI(G) = 6\left(\frac{4}{2}\right)^3 + (4an - 4)\left(\frac{6}{3}\right)^3 + (4an - (3a + 1))\left(\frac{12}{5}\right)^3 = 6(8) + (4an - 4)(8) + (4an - 3a - 1)\left(\frac{729}{64}\right)$$

$$= \frac{295}{64} + \frac{4964an}{64} - \frac{2187a}{64} = \frac{295}{64} + a\left(\frac{1241n}{16} - \frac{2187}{64}\right)$$

Propositions:

Indices	'a'	n=1	n=2	n=3	n=4	Indices	n=1	n=2	n=3	n=4
$\chi(G)$	2	12	$\frac{4}{\sqrt{16}} + \frac{4}{3} + 12$	$\frac{44}{3} + 4\sqrt{\frac{2}{3}}$	$\frac{12}{\sqrt{16}} + \frac{12}{3} + 12$	M3(G)	$\frac{3}{2}$	$\frac{47}{18}$	$\frac{67}{18}$	$\frac{29}{6}$
	3	$\frac{37}{3} + 2\sqrt{\frac{2}{3}}$	15 + 2√6	$\frac{53}{3} + 10\sqrt{\frac{2}{3}}$	$\frac{61}{3} + 14\sqrt{\frac{2}{3}}$		$\frac{41}{18}$	$\frac{9}{2}$	$\frac{121}{18}$	$\frac{161}{18}$
	4	$\frac{1}{3}(38 + 4\sqrt{6})$	$\frac{10}{3}(5 + \sqrt{6})$	$\frac{2}{3}(31 + 8\sqrt{6})$	$\frac{2}{3}(37 + 11\sqrt{6})$		$\frac{55}{18}$	$\frac{115}{18}$	$\frac{175}{18}$	$\frac{235}{18}$
GA(G)	2	6	$\frac{10}{5} + \frac{8\sqrt{6}}{5}$	$\frac{14}{5} + \frac{16\sqrt{6}}{5}$	$\frac{18}{5} + \frac{24\sqrt{6}}{5}$	IS(G)	6	$\frac{84}{5}$	$\frac{138}{5}$	$\frac{192}{5}$
	3	$\frac{1}{5}(35 + 8\sqrt{6})$	$\frac{3}{5}(25 + 8\sqrt{6})$	23 + 8√6	$\frac{1}{5}(155 + 56\sqrt{6})$		$\frac{123}{10}$	$\frac{339}{10}$	$\frac{111}{2}$	$\frac{771}{10}$
	4	$\frac{8}{5}(5 + 2\sqrt{6})$	4(5 + 2√6)	$\frac{32}{5}(5 + 2\sqrt{6})$	$\frac{44}{5}(5 + 2\sqrt{6})$		$\frac{93}{5}$	51	$\frac{417}{5}$	$\frac{579}{5}$

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SCI(G)	2	3	$\frac{4\sqrt{5}}{5} + \frac{2\sqrt{6}}{3} + 3$	$\frac{8\sqrt{5}}{5} + \frac{4\sqrt{6}}{3} + 3$	$\frac{2\sqrt{6}}{3} + \frac{5}{5}(5 + 4\sqrt{5})$	Alb(G)	0	4	8	12
	3	$\frac{1}{\sqrt{6}} + \frac{1}{5}(15 + 4\sqrt{5})$	$\frac{12}{\sqrt{5}} + \frac{3}{2}(2 + \sqrt{6})$	$\frac{4\sqrt{5}}{6} + \frac{17\sqrt{6}}{6} + 3$	$\frac{25}{\sqrt{6}} + \frac{1}{5}(15 + 28\sqrt{5})$		4	12	20	28
	4	$3 + \sqrt{\frac{2}{3}} + \frac{8}{\sqrt{5}}$	$3 + 7\sqrt{\frac{2}{3}} + \frac{4}{\sqrt{5}}$	$3 + 13\sqrt{\frac{2}{3}} + \frac{32}{\sqrt{5}}$	$3 + 19\sqrt{\frac{2}{3}} + \frac{44}{\sqrt{5}}$		8	20	32	42
HI(G)	2	3	$\frac{47}{9}$	$\frac{67}{9}$	$\frac{29}{3}$	ABC(G)	$3\sqrt{2}$	$\frac{8}{3} + 5\sqrt{2}$	$\frac{16}{3} + 7\sqrt{2}$	$8 + 9\sqrt{2}$
	3	$\frac{41}{9}$	9	$\frac{121}{9}$	$\frac{161}{9}$		$\frac{2}{3} + 5\sqrt{2}$	$6 + 9\sqrt{2}$	$\frac{34}{3}$	$\frac{50}{3} + 17\sqrt{2}$
	4	$\frac{55}{9}$	$\frac{115}{9}$	$\frac{175}{9}$	$\frac{235}{9}$		$\frac{4}{3} + 7\sqrt{2}$	$\frac{28}{3} + 13\sqrt{2}$	$\frac{52}{3} + 19\sqrt{2}$	$\frac{76}{3} + 25\sqrt{2}$
M1(G)	2	24	68	112	156	SD(G)	12	$\frac{86}{3}$	$\frac{136}{3}$	62
	3	50	138	256	314		$\frac{68}{3}$	56	$\frac{268}{3}$	$\frac{368}{3}$
	4	76	208	340	472		$\frac{100}{3}$	$\frac{250}{3}$	$\frac{400}{3}$	$\frac{550}{3}$

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M2(G)	2	24	84	144	204	AZI(G)	48	$\frac{2009}{16}$	$\frac{1625}{8}$	$\frac{4491}{16}$
	3	57	177	297	417		$\frac{5849}{64}$	$\frac{15777}{64}$	$\frac{25705}{64}$	$\frac{35633}{64}$
	4	90	270	450	630		$\frac{4313}{32}$	$\frac{11759}{32}$	$\frac{19205}{32}$	$\frac{26651}{32}$

VII. CONCLUSION

We present the generalized exact expression of the vertex degree based topological indices Randic, Geometric – Arithmetic, Sum Connectivity, Harmonic, First Zagreb, Second Zagreb, Second Modified Zagreb, Inverse Sum, Alberston, Atom – bond Connectivity, Symmetric – Division index and Augmented Zagreb indices for Linear Phynelene. The results obtained in our paper illustrated the promising application prospects in chemical and pharmacy engineering.

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